QUARK CONFINEMENT IN THE ANALYTIC APPROACH TO QCD

A. V. Nesterenko
Physical Department, Moscow State University, Moscow, 119899, Russia
E-mail: nesterav@thsun1.jinr.ru

In the early 60's in the papers [1,2] the so-called "Analytic approach" to quantum field theory was proposed. Its basic idea is the explicit imposition of the causality condition, which implies the requirement of the analyticity in the Q^2 variable for the relevant physical quantities. This approach was successfully applied to the analytization of the perturbative series when calculating the QCD observables [3]. It leads to the elimination of the unphysical singularities, to the higher loop correction stability and to a weak scheme dependence. However, the Q^2-evolution of some QCD observables (for instance, the structure function moments) is intimately tied with the solution of the renormalization group (RG) equation. Our task here is to involve the analytization procedure into the RG formalism more profoundly.

Let us consider the RG equation of a quite general form for quantity $\mathsf{A}(Q^2)$. At the one-loop level this equation and its solution read

$$\frac{d\ln\mathsf{A}(Q^2)}{d\ln Q^2} = \gamma\,\widetilde{\alpha}_{\rm s}^{(1)}(Q^2) \qquad \Longrightarrow \qquad \mathsf{A}(Q^2) = \mathsf{A}(Q_0^2)\left[\frac{\widetilde{\alpha}_{\rm s}^{(1)}(Q_0^2)}{\widetilde{\alpha}_{\rm s}^{(1)}(Q^2)}\right]^{\gamma}. \qquad (1)$$

Here γ is the corresponding anomalous dimension, $\widetilde{\alpha}_{\rm s}^{(1)}(Q^2) = \ln^{-1}(Q^2/\Lambda^2)$ is the one loop perturbative running coupling. Obviously, the solution of Eq. (1) has unphysical singularities in the physical region $Q^2 > 0$. However, in many interesting cases the quantity $\mathsf{A}(Q^2)$ must have correct analytic properties in the Q^2 variable (namely, there is the only cutoff $Q^2 \le 0$). One can demonstrate this proceeding only from the first principles.

At first sight, we come to the contradiction. Indeed, the left-hand side of the standard RG equation (1) has no unphysical singularities in the $Q^2 > 0$ region, while its right-hand side has pole-type singularity at the point $Q^2 = \Lambda^2$. The account of the higher loop contributions just introduces the additional unphysical singularities of the cut type in the physical region $Q^2 > 0$ and hence does not solve the problem. But there is no real contradiction here. This situation arose with the perturbative expansion of the anomalous dimension in the right-hand side of the RG equation.

In order to improve the situation, we propose to use the following method [4]. Before solving the RG equation (1) one should analytize its right-hand side as

a whole for restoring correct analytic properties. This prescription leads to the analytized RG equation, which, at the one-loop level, takes the form

$$\frac{d\ln \mathsf{A}(Q^2)}{d\ln Q^2} = \gamma\, \widetilde{\alpha}_{\rm an}^{(1)}(Q^2) \qquad \Longrightarrow \qquad \mathsf{A}(Q^2) = \mathsf{A}(Q_0^2)\left[\frac{{}^{\rm N}\widetilde{\alpha}_{\rm an}^{(1)}(Q_0^2)}{{}^{\rm N}\widetilde{\alpha}_{\rm an}^{(1)}(Q^2)}\right]^{\gamma}. \quad (2)$$

Here $\widetilde{\alpha}_{\rm an}^{(1)}(Q^2) = \ln^{-1}(Q^2/\Lambda^2) + (1 - Q^2/\Lambda^2)^{-1}$ is the perturbative running coupling analytized by making use of the Shirkov–Solovtsov prescription [3]. So, the solution of the analytized RG equation is expressed in terms of the new analytic running coupling which at the one-loop level has the form [4]:

$${}^{\rm N}\alpha_{\rm an}^{(1)}(Q^2) = \frac{4\pi}{\beta_0}\frac{z-1}{z\ln z}, \quad z = \frac{Q^2}{\Lambda^2}. \quad (3)$$

The distinctive feature of this running coupling, that plays the crucial role in the framework of our consideration, is its infrared (IR) enhancement. It is worth noting that such a behavior of the invariant charge is in agreement with the Schwinger–Dyson equations (see discussion in Ref. [5]), and, as it will be demonstrated further, provides the quark confinement *without invoking any additional assumption.*

At the higher loop levels there is only the integral representation for ${}^{\rm N}\alpha_{\rm an}(Q^2)$. Figure 1 shows the new analytic running coupling computed at the one-, two-, and three-loop levels. The essential merits of the running coupling ${}^{\rm N}\alpha_{\rm an}$ are the following (see Ref. [4] for the details): It has no unphysical singularities at any loop level; It is stable with respect to both the higher loop corrections and the scheme dependence; Its singularity at the point $Q^2 = 0$ is of the universal type at any loop level; Its IR behavior is in agreement with the Schwinger–Dyson equations.

Let us turn to obtaining the quark-antiquark potential. We proceed from the standard expression [6,7] for this potential in terms of the running coupling $\alpha(q^2)$,

$$V(r) = -\frac{16\pi}{3}\int_0^{\infty} \frac{\alpha(q^2)}{q^2}\frac{e^{i\mathbf{qr}}}{(2\pi)^3}\, d\mathbf{q}. \quad (4)$$

For the construction of the new interquark potential ${}^{\rm N}V(r)$ we shall use the new analytic running coupling (3). Upon the integration over the angular variables and some calculations (see Ref. [4] for the details) one can present this potential at large distances in the following way:

$${}^{\rm N}V(r) \simeq \frac{8\pi}{3\beta_0}\Lambda\cdot\frac{R}{2\ln R}, \quad R = \Lambda r, \quad R \to \infty. \quad (5)$$

Thus the new analytic running coupling ${}^{\mathrm{N}}\alpha_{\mathrm{an}}(q^2)$ (see Eq. (3)) leads to the rising quark-antiquark potential ${}^{\mathrm{N}}V(r)$ which can, in principle, describe the quark confinement. At the same time the behavior of the potential ${}^{\mathrm{N}}V(r)$ when $r \to 0$ has the standard form [7] determined by the asymptotic freedom: ${}^{\mathrm{N}}V(r) \simeq \Lambda(R \ln R)^{-1}$. It can be shown also that the obtained result is stable with respect to both the higher loop corrections and the scheme dependence [4].

For the practical use of the new potential it is worth obtaining a simple explicit expression that approximates it sufficiently well. For this purpose one can use, for instance, the approximating function $U(r)$ (see Ref. [4]). Of course, this function is not the unique one. Nevertheless, the comparison of $U(r)$ with the phenomenological Cornell potential shows their almost complete coincidence (see Fig. 2). A rough estimation of parameter Λ in the course of this fitting gives $\Lambda \simeq 500$ MeV. Estimation of Λ from the gluon condensate for the ${}^{\mathrm{N}}\alpha_{\mathrm{an}}$ has been performed recently and gives the close value, $\Lambda = 530 \pm 50$ MeV.

Thus, we infer that combination of the RG formalism with the Analytic approach, in principle, enables one to obtain the confining quark-antiquark potential.

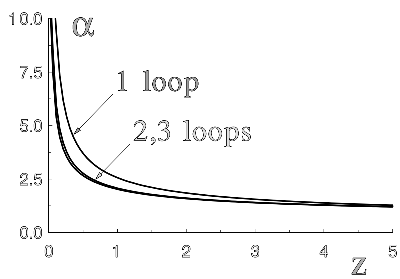

Figure 1: The normalized new analytic running coupling at the one-, two-, and three-loop levels.

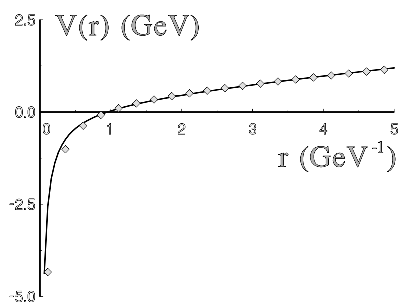

Figure 2: Comparison of the $U(r)$ (solid curve) and the Cornell ($\diamond$) potentials; $\Lambda = 530$ MeV, $n_f = 5$.

References

1. P. J. Redmond, *Phys. Rev.* **112**, 1404 (1958).
2. N. N. Bogoliubov *et al.*, *Sov. Phys. JETP* **37**, 574 (1960).
3. D. V. Shirkov and I. L. Solovtsov, *Phys. Rev. Lett.* **79**, 1209 (1997).
4. A. V. Nesterenko, *Phys. Rev.* D **62**, 094028 (2000); `hep-ph/9912351`.
5. A. I. Alekseev and B. A. Arbuzov, *Mod. Phys. Lett.* A **13**, 1747 (1998).
6. N. Brambilla and A. Vairo, `hep-ph/9904330`.
7. J. L. Richardson, *Phys. Lett.* B **82**, 272 (1979).